Finite Element        ثلاثة مني
                      c.۱۲ - c.۱۵



$$A = \pi r^2 \qquad \Rightarrow \qquad A = \sum \left( \frac{1}{2} R^2 \sin \theta \right)$$

with F.E.M.



with F.E.M.

* يتم تقسيم الشكل الاصلي الى اشكال أصغر لنفض نماذج اقرب
  الى الحقيقة

* حيث يصعب حل الشكل الاصلي فيقسم ليعطى حل مبسط وتريب
  من القيمة الحقيقية
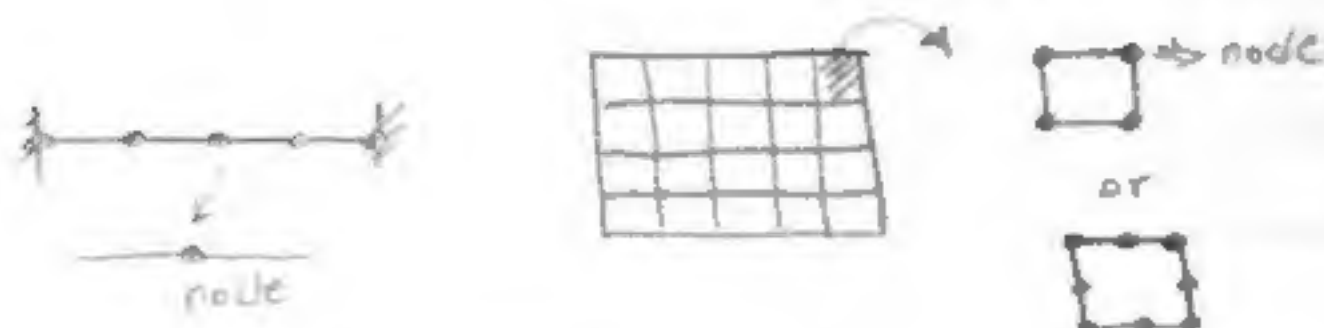
* كلما زادت عدد الاجزاء    كلما اقترب الحل من الواقع

* Feilds used F.E.M. "Finite element method"

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1- Mechanical engineering
2- structural analysis
3- Thermal fluid Mechanics
4- Electro magnetics
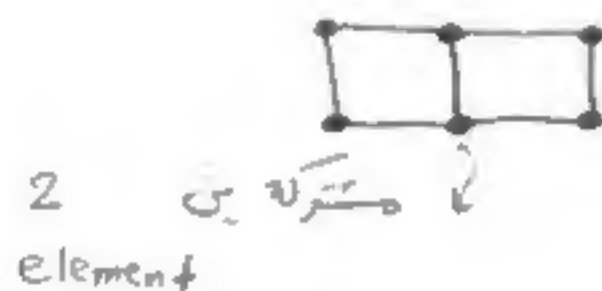5- geo mechanics
6- Bio mechanics

\* steps of F.E.M.

1 — Divid structure into pieces

"elements with nodes"



node

or



→ node

كمان بعد node كله الحد بربط معاه الحد المعلمة

2 — Connect elements of node



2 هنا بيكون فيه
element

( بربط element هنا )

3 — Describe behavior of physical quantity of each element

تحديد سلوك ده حدوث على كل element

4 — Solve system of equations "stiffness method"

\* Types of element

① 1-D element

1 •———————• 2

ex spring , Truss, beam, pipe
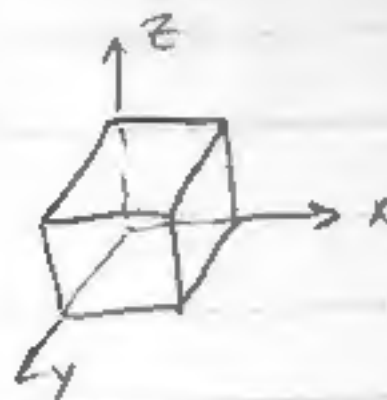
② 2-D element

    _ex_ membrane, plate, shell

③ 3-D element

    _ex_ 8-node element (solid)

         6-node element (pnsm)

\* Conditions To apply F.E.M

1- static load is "slow steady-rate of loading"

      تحميل ثابت و سرعة بطيئة

2- elastic material

      ? قبل ان yeild

3- small deformation

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## Truss / Bar element

- 2 node = $i, j$
- each node has 1 Dof

    $U_x$

∴ we have $U_{xi}, U_{xj}$

- axial force element

الاجهادات المحورية

strain - disp. ⟹ $\varepsilon = \dfrac{du}{dx} = \dfrac{\Delta}{L}$

stress-disp $\Rightarrow$ $\sigma = E \varepsilon = \dfrac{E \Delta}{L}$

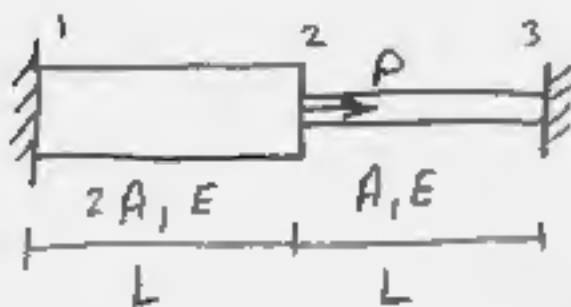$\because \sigma = \dfrac{F}{A}$

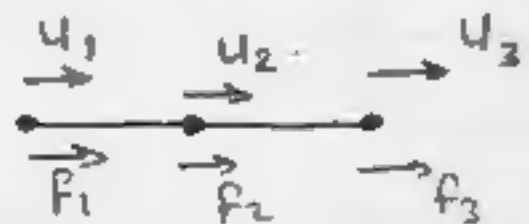$\therefore F = \dfrac{EA}{L} \cdot \Delta = K \Delta$

$\hookrightarrow K = \begin{bmatrix} \dfrac{EA}{L} & -\dfrac{EA}{L} \\ -\dfrac{EA}{L} & \dfrac{EA}{L} \end{bmatrix}$

$\therefore \begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \dfrac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$

Ex 1



Find stress

(sol) element 1

$k_1 = \dfrac{2EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$

element 2

$K_2 = \dfrac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} ② \\ ③ \end{matrix}$

$K_{tot.} = \dfrac{EA}{L} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \dfrac{EA}{L} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\therefore \quad u_1 = u_3 = zero$$
$$f_2 = P$$

$$\therefore \quad P = \frac{EA}{L} \left[ -2*0 + 3u_2 - 1*0 \right] = 3\frac{EA}{L} u_2$$
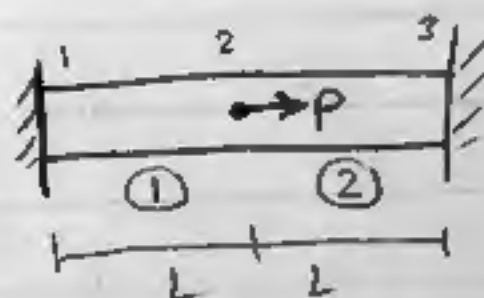
$$u_2 = \frac{PL}{3EA}$$

$$\therefore \quad u = \frac{PL}{3EA} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \quad \Rightarrow \quad \sigma = \frac{E\Delta}{L}$$

$$\sigma_1 = 2E * \frac{u_2 - u_1}{L} = 2E * \frac{\frac{PL}{3EA} - 0}{L} = \frac{2P}{3A}$$

$$\sigma_2 = E * \frac{u_3 - u_2}{L} = E * \frac{0 - \frac{PL}{3EA}}{L} = -\frac{P}{3A}$$

EX 2



$E = 2*10^4 \ N/mm^2$

$A = 250 \ mm^2$

$P = 6 * 10^4 \ N$

$L = 150 \ mm$

$\Delta_3 = 1.2 \ mm$

find support reaction

$$k_1 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad k_2 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\therefore \quad u_3 = 1.2 \qquad\qquad , \quad u_1 = 0$$

$$f_2 = P = 6*10^4$$

$$\therefore \quad 6*10^4 = \frac{2*10^4 *250}{150}(-1*0 + 2u_2 - 1*1.2)$$

$$u_2 = 1.5\,mm$$

$$f_1 = -5*10^4$$
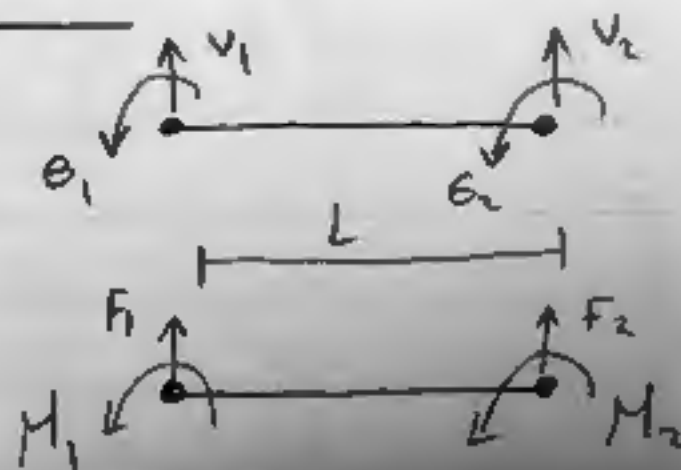$$f_3 = -1*10^4$$

بالتعويض
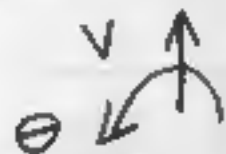
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## Beam element

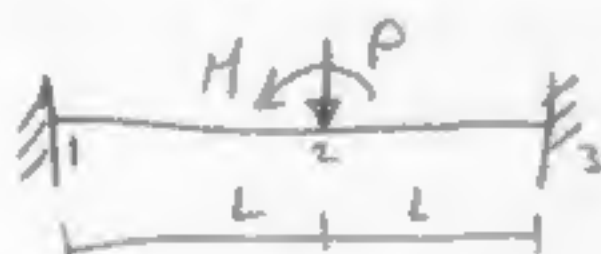- each node have 2 DoF

V ↑
Θ ↙ ↑

$$M = EI\frac{d^2v}{dx^2}$$

$$\sigma = \frac{-My}{I}$$

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

Ex 3



Find deflection & rotation at center node
reactions at ends

$$k_1 = \frac{EI}{L^3}$$
$$k_2 = \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix} = \frac{EI}{L^3} \qquad \begin{Bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \\ V_3 \\ \theta_3 \end{Bmatrix}$$
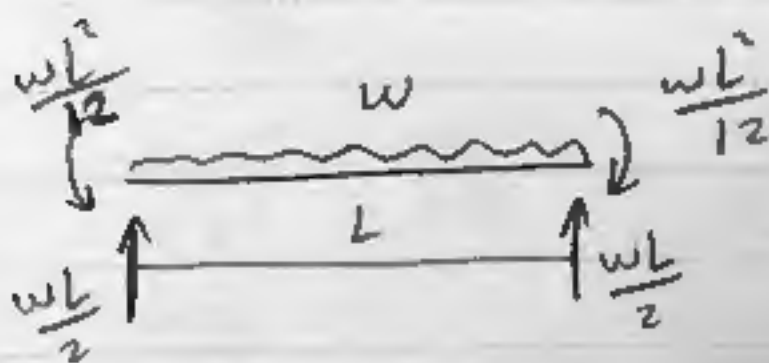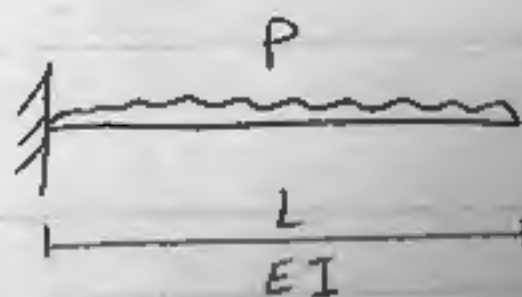
$$V_1, \theta_1, V_3, \theta_3 = zero \qquad \text{بما}$$
$$M_2 = M$$
$$F_2 = -P$$

✱ ✱



$\frac{wL^2}{12}$    $w$    $\frac{wL^2}{12}$    ( Fixed end moment )

$L$

$\frac{wL}{2}$    $\frac{wL}{2}$

**Ex 4**



$P$

$L$

$EI$

Find deformation of free end

$$k = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\begin{aligned} &\frac{PL}{2} = \\ &-\frac{PL^2}{12} = \end{aligned} \quad \begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \end{Bmatrix} = \quad \begin{Bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \end{Bmatrix} \begin{aligned} &= 0 \\ &= 0 \end{aligned}$$

$$+\frac{12EI}{L^3} V_2 - \frac{6EI}{L^2} \theta_2 = +\frac{PL}{2}$$

$$-\frac{6EI}{L^2} V_2 + \frac{4EI}{L} \theta_2 = -\frac{PL^2}{12}$$

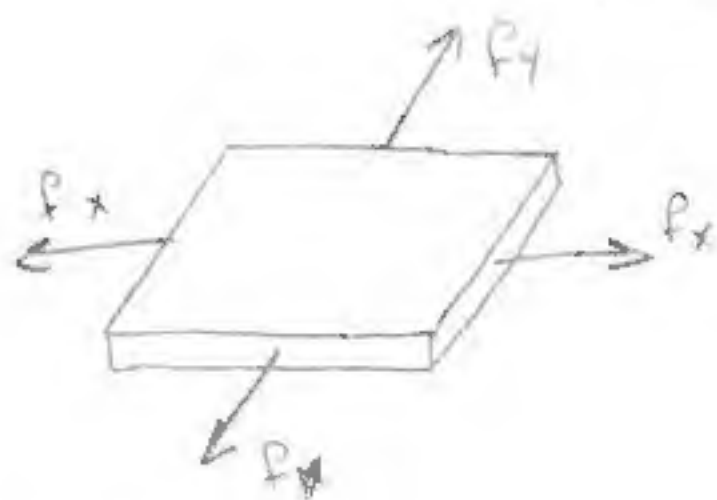$$\therefore \; \theta_2 = -\frac{PL^3}{6EI} \qquad\qquad V_2 = -\frac{PL^4}{8EI}$$

$$\therefore \; P_1 = -12\frac{EI}{L^3}\left(-\frac{PL^4}{8EI}\right) + \frac{6EI}{L^2}\left(-\frac{PL^3}{6EI}\right)$$

$$= +\frac{PL}{2}$$
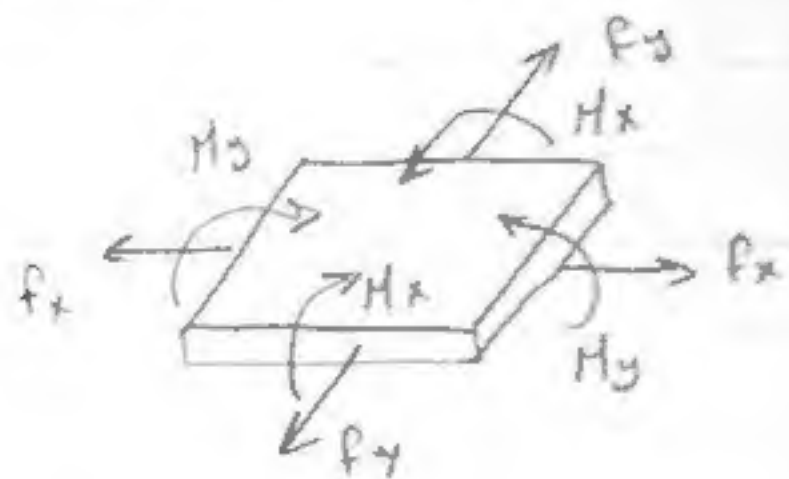
$$M_1 = -\frac{6EI}{L^2}\left(-\frac{PL^4}{8EI}\right) + \frac{2EI}{L}\left(-\frac{PL^3}{6EI}\right)$$

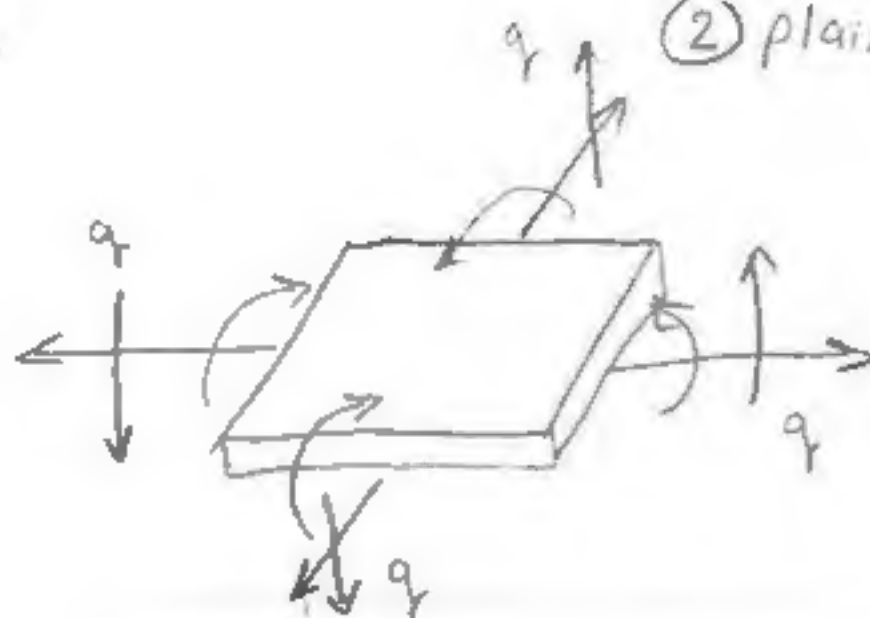$$= \frac{PL^2}{12}$$

# Shell element



① Memberane

② plain stress



③ plate bending

# Solid element



① Hexahedral (Brick)
   8 nodes
                    (linear)

② quadratic
   20 nodes

strain - disp.

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$